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ADMISSIBLE DIAGRAMS FOR $G(m,1,n)$

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Abstract. In this paper, we show that every conjugacy class of the imprimitive complex reflection group $G(m,1,n)$ can be represented by an admissible diagram. For this, we introduce a length function for elements of $G(m,1,n)$ and study its properties. This then allows us to establish the admissible diagram for each conjugacy class of $G(m,1,n)$. The corresponding results for Weyl groups and their conjugacy classes are well known.

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1. Introduction

The main objective of this paper is to show that there is a one-to-one correspondence between conjugacy classes in the imprimitive complex reflection group $G(m,1,n)$ and admissible diagrams whose connected components all have type A or B. The group $G(m,1,n)$ can be viewed as the generalized symmetric group and its conjugacy classes can be found in Kerber [8]. To establish an admissible diagram for every conjugacy class of $G(m,1,n)$, in the second section we define a length function for $G(m,1,n)$ and study its properties. The corresponding results for Weyl groups were studied by Carter [5].

We first give the basic notation and state some results which are required later. We refer the reader to [3] and [6] for most of the undefined terminology and quoted results. As a convention, throughout this paper, we assume that ξ is a primitive m-th root of unity.

Let $V = \mathbf{C}^n$ be the complex vector space of dimension n with standard unitary inner product $(\cdot,\cdot)$ and the standard basis $\{e_1, e_2, \ldots, e_n\}$. A *reflection* in V is a linear transformation of V of finite order with exactly $n-1$ eigenvalues equal to 1. A *reflection group* G in V is a finite group generated by reflections in V.

For each non-zero vector $\alpha \in V$, let w_α be a reflection in V of order $m > 1$. Then there is a primitive m-th root of unity ξ such that

$$w_\alpha(v) = v - (1-\xi)\frac{(v,\alpha)}{(\alpha,\alpha)}\,\alpha$$

for all $v \in V$. Thus $w_\alpha(\alpha) = \xi\alpha$ and $w_\alpha(v) = v$ if $v \in \langle\alpha\rangle^\perp$, where $\langle\alpha\rangle^\perp$ is the orthogonal complement of $\langle\alpha\rangle$ with respect to the given unitary inner product. Define $o_G :\ V \to \mathbf{N}$ by $o_G(v) = |G_{\langle v\rangle^\perp}|$ $(v \in V)$. Then $o_G(v) > 1$ if and only if v is a root of G. In this case, $o_G(v)$ is the order of the cyclic group generated

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by the reflections in G with root v. If α is a root of G, then the number $o_G(\alpha)$ is called the *order* of α.

Let $G(m,1,n)$ be the *imprimitive complex reflection group* in V generated by reflections of order 2 with roots $e_1-e_2,\ e_2-e_3,\ \ldots,\ e_{n-1}-e_n$ and the reflection of order m with root e_n. A root system for $G(m,1,n)$ may be defined as follows (see [6]). Let $\mu_m = \{\xi^l \mid l \in \mathbf{N},\ \xi$ be a primitive m-th root of unity$\}$. Put

$$R(m,1,n) = \mu_m \left\{\pm(e_i - \xi^l e_j),\ e_k \mid i,\ j,\ k,\ l \in \mathbf{N},\ i \neq j,\ 1 \leq i,\ j,\ k \leq n\right\}$$

and let $f_{m,1,n}:\ R(m,1,n) \to \mathbf{N}\backslash\{1\}$ be defined by

$$f_{m,1,n}(\alpha) = \begin{cases} m & \text{if } \alpha \in \mu_m\{e_k \mid 1 \leq k \leq n\}, \\ 2 & \text{otherwise;} \end{cases}$$

then we have that $\Phi = \Phi(m,1,n) = (R(m,1,n), f_{m,1,n})$ is a root system with $W(\Phi) = G(m,1,n)$. The group $G(m,1,n)$ has the following presentation (see [7]):

$$G(m,1,n) = \langle r_1,\ldots,r_{n-1},w_1,\ldots,w_n \mid r_i^2 = (r_i r_{i+1})^3 = (r_i r_j)^2 = 1,\ |i-j| \geq 2, \\ w_i^m = 1,\ w_i w_j = w_j w_i,\ r_i w_i = w_{i+1} r_i,\ r_i w_j = w_j r_i,\ j \neq i,\ i+1\rangle.$$

2. The Length Function

Let $W = G(m,1,n)$ denote the imprimitive reflection group corresponding to $\Phi = \Phi(m,1,n)$. In this section we introduce a length function for W and study its properties.

Now each element w in W can be expressed as a product of reflections $w = w_{a_1}^{s_1} \ldots w_{a_k}^{s_k}$, where $a_i \in \Phi$ and $s_i \in \{1,\ldots,m-1\}$. The *length* of w, denoted by $l(w)$ is the smallest value of $\sum_{i=1}^{k} s_i$ in any such expression for w. (Here if $o_W(a_i) = 2$, then $s_i = 1$, and if $o_W(a_i) = m$, then $s_i \in \{1,\ldots,m-1\}$.) By convention, $l(1) = 0$. Clearly, $l(w) = 1$ if and only if $w = w_a$ where $a \in \Phi$. It is also clear that if $w = w_a^s$ with $o_W(a) = m$ and $s \in \{1,\ldots,m-1\}$, then $l(w) = s$. We say that w is a product of k reflections if $l(w) = \sum_{i=1}^{k} s_i$.

Any element $\sigma \in W$ can be written uniquely (up to reordering) as the product of disjoint cycles $\sigma = \theta_1 \ldots \theta_t$, where

$$\theta_i = \begin{pmatrix} b_{i1} & b_{i2} & \ldots & b_{ik_i} \\ \xi^{s_{i1}} b_{i2} & \xi^{s_{i2}} b_{i3} & \ldots & \xi^{s_{ik_i}} b_{i1} \end{pmatrix},$$

$b_{ij} \in \{1,\ldots,n\},\ s_{ij} \in \{1,\ldots,m\},\ k_i$ is the length of the cycle $\theta_i,\ i = 1,\ldots,t$. Let $f(\theta_i) = \sum_{j=1}^{k_i} s_{ij}$, and put $f(\sigma) = \sum_{i=1}^{t} f(\theta_i)$.

Now, define $a_{pq}(\sigma)$ to be the number of cycles θ_i of σ of length q such that $f(\theta_i) \equiv p \pmod{m}$ for $1 \leq p \leq m,\ 1 \leq q \leq n$. The $m \times n$ matrix $(a_{pq}(\sigma))$ is called the *type* of σ, denoted by $Ty(\sigma)$ (see [9]). Then $\sigma,\ \pi \in W$ are conjugate in W if and only if $Ty(\sigma) = Ty(\pi)$ (see [8]).

Lemma 2.1. *If $\sigma,\ \pi \in W$ are conjugate in W, then $l(\sigma) = l(\pi)$.*

Proof. Let $\sigma = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k}$, where $a_i \in \Phi$ and $s_i \in \{1, \ldots, m-1\}$. Since σ is conjugate in W to π, $\pi = w\sigma w^{-1}$ for some $w \in W$. But $w\sigma w^{-1} = w_{b_1}^{s_1} \ldots w_{b_k}^{s_k}$ where $b_i = w(a_i)$ implies that $l(\sigma) = \sum_{i=1}^{k} s_i = l(\pi)$. □

The above lemma says that two conjugate elements in W have the same length and each of them is also the product of the same number of reflections.

The lemma below is a well-known property of reflection groups (see [10]).

Lemma 2.2. *Let G be a reflection group in an n-dimensional complex vector space V. If $g \in G$ and U is the subspace of V composed of all vectors fixed by g, then g is a product of the reflections corresponding to the roots in the orthogonal complement $U^\perp$ of U.*

Lemma 2.3. *Let $w \in W$. Then $l(w)$ is a sum of the powers of eigenvalues of w on V which are not equal to* 1.

Proof. Suppose that $l(w) = \sum_{i=1}^{k} s_i$. Then w is a product of k reflections and has an expression of the form $w = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k}$, where $a_i \in \Phi$ and $s_i \in \{1, \ldots, m-1\}$. Now, let H_{a_i} be the reflecting hyperplane of a_i in V and let

$$H = H_{a_1} \cap H_{a_2} \cap \cdots \cap H_{a_k}.$$

Then w acts trivially on H and $\dim H \geq n-k$. Thus w has at least $n-k$ eigenvalues equal to 1, and so at most k eigenvalues $\xi^{s_1}, \xi^{s_2}, \ldots, \xi^{s_k}$ which are not equal to 1, by the definition of a reflection. Therefore, the sum of the powers of these eigenvalues is not more than $l(w)$.

Conversely, suppose w has k eigenvalues $\xi^{s_1}, \xi^{s_2}, \ldots, \xi^{s_k}$ which are not equal to 1, where $s_i \in \{1, \ldots, m-1\}$. Let U be the subspace of V composed of all vectors fixed by w, and $U^\perp$ be the orthogonal subspace. Then at once $\dim U = n-k$ and $\dim U^\perp = k$, and by Lemma 2.2 w is a product of the reflections corresponding to the roots in $U^\perp$.

Suppose that w fixes some vector in V. Then $k < n$ and so $\dim U^\perp < \dim V$. The roots in $U^\perp$ form a root system in the subspace they generate which has dimension less than n, and w is an element of the reflection group associated with this root system. Thus, by induction, w is a product of at most k reflections, i.e., $w = w_{a_1}^{s_1} \ldots w_{a_k}^{s_k}$, and so $l(w) \leq \sum_{i=1}^{k} s_i$. □

An expression $w_{a_1}^{s_1} \cdots w_{a_k}^{s_k} \in W$ is called *reduced* if $l(w_{a_1}^{s_1} \ldots w_{a_k}^{s_k}) = \sum_{i=1}^{k} s_i$.

Lemma 2.4. *Let $a_1, \ldots, a_k \in \Phi$ and $s_i \in \{1, \ldots, m-1\}$ for $i = 1, \ldots, k$. Then $w_{a_1}^{s_1} \ldots w_{a_k}^{s_k}$ is reduced if and only if $a_1, \ldots, a_k$ are linearly independent.*

Proof. Let $w = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k}$. Suppose that the expression is reduced. Then by Lemma 2.3, w has k eigenvalues not equal to 1, and so

$$\dim(H_{a_1} \cap H_{a_2} \cap \cdots \cap H_{a_k}) = n - k.$$

(Here, the dimension cannot be larger, since w acts as the identity on this subspace.) Thus it follows that the roots $a_1, \ldots, a_k$ are linearly independent.

Conversely, suppose that the roots $a_1, \ldots, a_k$ are linearly independent. Now, consider the subspace $im(w-1)$, and choose a vector $v_1 \in V$ such that

$$v_1 \in H_{a_2} \cap \cdots \cap H_{a_k} \quad \text{but} \quad v_1 \notin H_{a_1}.$$

Then $w(v_1) - v_1$ is a non-zero multiple of a_1. Thus $a_1 \in im(w-1)$. Now, select once again a vector $v_2 \in V$ with

$$v_2 \in H_{a_3} \cap \cdots \cap H_{a_k} \quad \text{but} \quad v_2 \notin H_{a_2}.$$

Then $w(v_2) - v_2 = \alpha a_1 + \beta a_2$, where $\alpha,\ \beta \in \mathbf{C}$ and $\beta \neq 0$. Hence $a_2 \in im(w-1)$. Repeating this argument will eventually show that $a_1, \ldots, a_k \in im(w-1)$, and so $\dim im(w-1) = k$.

Then w is reduced, for if w has a shorter expression $w = w_{b_1}^{\rho_1} \ldots w_{b_l}^{\rho_l}$ with $l < k$ and $\rho_i \in \{1, \ldots, m-1\}$, then every element of $im(w-1)$ can be written as a linear combination of $b_1, \ldots, b_l$ and so $\dim im(w-1) < k$, which is a contradiction. Furthermore, if w has an expression $w = w_{a_1}^{r_1} \ldots w_{a_k}^{r_k}$ with $r_i \leq s_i$, then $w_{a_k}^{\varrho_k} \ldots w_{a_1}^{\varrho_1} w = 1$ and $w_{a_k}^{\varrho_k} \ldots w_{a_1}^{\varrho_1} w_{a_1}^{r_1} \ldots w_{a_k}^{r_k} \neq 1$ where $\varrho_i = o_W(a_i) - s_i$ $(1 \leq i \leq k)$, a contradiction. □

3. Admissible Diagrams

Let $\Phi(m,p,n)$ $(p = 1,\ m)$ be an imprimitive root system with simple system $\pi(m,p,n) = (B, \theta)$, where

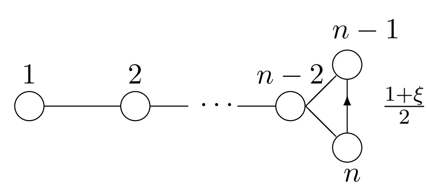
$$B = \begin{cases} \{\alpha_i = e_i - e_{i+1} \ (i = 1,\ \ldots,\ n-1),\ \alpha_n = e_n\} & \text{if } p = 1, \\ \{\beta_i = e_i - e_{i+1} \ (i = 1,\ \ldots,\ n-1),\ \beta_n = e_{n-1} - \xi e_n\} & \text{if } p = m. \end{cases}$$

Then the Cohen diagrams for $\Phi(m,1,n)$ and $\Phi(m,m,n)$ are respectively

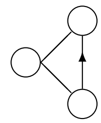
B_n^m :

where the node corresponding to α_i $(i = 1, \ldots, n)$ is denoted by i and

D_n^m :



where the node corresponding to β_i $(i = 1, \ldots, n)$ is denoted by i.

A *web* is a graph of the form  $\frac{1+\xi^s}{2}$, where $s \in \{1, \ldots, m-1\}$.

Any element $w = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k} \in W$ with $l(w) = \sum_{i=1}^{k} s_i$ can be decomposed as follows (see [1] or [2]):

$$w = \tau w_{a_{i+1}}^{s_{i+1}} \cdots w_{a_k}^{s_k}, \quad \text{where} \quad \tau = w_{a_1} \cdots w_{a_i} \in W(A_{n-1}).$$

For each such decomposition of w, we define the graph Γ as having k nodes, one corresponding to each of the roots $a_1, \ldots, a_k$ with the value $o_W(a_i)$. The nodes corresponding to distinct roots $a_i,\ a_j$ are joined by a bond of weight $(a_i,\ a_j)$. If $o_W(a_i) = 2$, then the number 2 in the node corresponding to the root a_i is omitted, as in Cohen [6].

If $w \in W$ has a decomposition with graph Γ, then any conjugate of w also has a decomposition with graph Γ. For if $w = w_{a_1} \cdots w_{a_i} w_{a_{i+1}}^{s_{i+1}} \cdots w_{a_k}^{s_k}$, where $w_{a_1} \cdots w_{a_i} \in W(A_{n-1})$, then we have $w' w w'^{-1} = w_{b_1} \cdots w_{b_i} w_{b_{i+1}}^{s_{i+1}} \cdots w_{b_k}^{s_k}$, where $b_j = w'(a_j)$ for $j = 1, \ldots, k$.

Therefore we say that the graph Γ is associated with this conjugacy class. (Here, we assume that the conjugacy class containing the identity element is represented by the empty graph.) By Lemma 2.4 the nodes of Γ correspond to a set of linearly independent roots.

Now we can give our basic definition.

Definition 3.1. Let Γ be a graph, then Γ is called an *admissible diagram* if
(i) the nodes of Γ correspond to a set of linearly independent roots of Φ,
(ii) each subgraph of Γ which is a cycle is equivalent to a web.

(A subgraph of Γ in this context is a subset of the nodes, together with the bonds joining the nodes in the subset. A cycle is a graph in which each node is connected only to two other nodes.)

Lemma 3.2. *Every admissible diagram associated with a conjugacy class of W is the Cohen (Dynkin) diagram of some reflection subgroup of W.*

Proof. Let Γ be such a graph. Let J be a set of the roots corresponding to the nodes of Γ. Denote by $W(J)$ the group generated by all reflections $w_{a,o_W(a)}$ with $a \in J$, then $W(J)$ is a subgroup of W and so is a finite reflection group. Furthermore, J is linearly independent by the definition of an admissible diagram. Thus, by (4.2) of Cohen [6], Γ is a root graph.

Now, put $S = W(J)J$, define a map $g : S \to \mathbf{N}\backslash\{1\}$ by $g(a) = o_{W(J)}(a)$ for all $a \in S$, then the pair $\Psi = (S, g)$ is the pre-root system corresponding to J with $W(\Psi) = W(J)$ by 1.2 (ii) of Can [3]. By 1.2 (iii) of Can [3], the pair $\Psi = (S, g)$ is a root system and so is a subsystem of Φ. Thus, $W(\Psi)$ is the reflection group of Ψ, and so Γ is the Cohen (Dynkin) diagram of the reflection subgroup $W(\Psi)$ of W, as desired. □

Now, recall that Γ may be a union of disconnected graphs Γ_i, which, say, satisfy the following: if Γ_i contains no web, then Γ_i is either of type A or B, and if Γ_i does contain a web, then Γ_i must be of type D.

The present author [3] has presented an algorithm for obtaining graphs which are Cohen (Dynkin) diagrams of reflection subgroups of W without any reference to extended diagrams. In [4], we interpreted this algorithm as a computer program written by using the Maple symbolic computation system.

We now show that admissible diagrams can be used to parametrize the conjugacy classes of W.

Theorem 3.3. *Let $W = G(m,1,n)$. There is a one-to-one correspondence between conjugacy classes in W and admissible diagrams of the form*

$$\sum_{p=1}^{m} (B^{m_p}_{\lambda_1^{(p)}} + B^{m_p}_{\lambda_2^{(p)}} + \cdots + B^{m_p}_{\lambda_{s_p}^{(p)}}),$$

where

$$\sum_{p=1}^{m-1}\sum_{q=1}^{s_p} \lambda_q^{(p)} + \sum_{q=1}^{s_m}(\lambda_q^{(m)} + 1) = n \quad \text{and} \quad m_p = \frac{m}{(m,p)}\,,$$

where (m,p) is the greatest common divisor of m and p.

Proof. The elements of W operate on the orthonormal basis $e_1,\dots,e_n$ of V by permuting the basis vectors and multiplying arbitrary subsets of them by a power of ξ. Ignoring these multiples, each element w of W determines a permutation of $\{1,\dots,n\}$ which can be expressed in the usual way as a product of disjoint cycles. Let $(k_1k_2\cdots k_r)$ be such a cycle written as

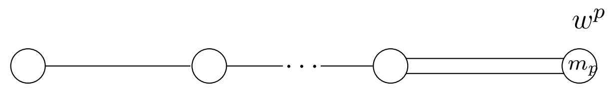
$$e_{k_1} \longrightarrow \xi^{p_1}e_{k_2} \longrightarrow \xi^{p_1+p_2}e_{k_3} \longrightarrow \cdots \longrightarrow \xi^{p_1+\cdots+p_{r-1}}e_{k_r} \longrightarrow \xi^{p_1+\cdots+p_r}e_{k_1},$$

where $p_i \in \{1,\dots,m\}$. The cycle $(k_1k_2\cdots k_r)$ is said to be a (ξ^p, r)-cycle, denoted by $[r^{\xi^p}]$, if $w^r(e_{k_1}) = \xi^p e_{k_1}$, where $\sum_{i=1}^{r} p_i \equiv p \pmod{m}$. Then the lengths of the cycles together with their values $\sum p_i$ determine the type of w, and two elements of W are conjugate if and only if they have the same type, as in Kerber [8]. Thus there is a one-to-one correspondence between conjugacy classes and types. Now, consider the (ξ^p, r)-cycle

$$e_1 \longrightarrow e_2 \longrightarrow \cdots \longrightarrow e_{r-1} \longrightarrow e_r \longrightarrow \xi^p e_1,$$

where $p \in \{1,\dots,m\}$. If $p = m$, then this can be expressed as the product of elements $(12)(23)\ \cdots\ (r-1\ r)$. These factors form a complete set of simple reflections of the Weyl subgroup of type A_{r-1}, and so this $(1,r)$- cycle, denoted by $[r]$, is represented by an admissible diagram A_{r-1}, as in type A_n (see Carter [5]). If $p \in \{1,\dots,m-1\}$, then this can be expressed as the product of elements $(12)(23)\ \cdots\ (r-1\ r)w_r^p$, where w_r^p changes e_r into $\xi^p e_r$ and fixes all other e_i. Thus these factors form a complete set of simple reflections of the reflection subgroup of type $B_r^{m_p}$, where $m_p = \frac{m}{(m,p)}$ where (m,p) is the g.c.d. of m and p, and so this (ξ^p, r)-cycle is represented by an admissible diagram

$B_r^{m_p}$ (r nodes) :



(This is a natural choice for the following reason. If for the group $G(m,1,n)$ we take $n=1$, then we have $C_m = G(m,1,1) = \langle w \mid w^m = 1\rangle$ which is the cyclic group of order m. We attach to each non-identity class (element) w^p $(1 \le p \le m-1)$ in C_m an admissible diagram

i.e., the admissible diagram depends on the order of the element.)

Now consider an arbitrary element of W, expressed as a product of disjoint (ξ^p, r)-cycles. Since disjoint cycles operate on orthogonal subspaces of V, the admissible diagram splits into connected components corresponding to the cycle decomposition, and so takes form

$$\sum_{p=1}^{m} (B^{m_p}_{\lambda_1^{(p)}} + B^{m_p}_{\lambda_2^{(p)}} + \cdots + B^{m_p}_{\lambda_{s_p}^{(p)}}),$$

where

$$\sum_{p=1}^{m-1} \sum_{q=1}^{s_p} \lambda_q^{(p)} + \sum_{q=1}^{s_m} (\lambda_q^{(m)} + 1) = n \quad \text{and} \quad m_p = \frac{m}{(m,p)},$$

where (m,p) is the g.c.d. of m and p, as desired. □

Remark 3.4. Now, define m partitions $\lambda^{(1)}, \ldots, \lambda^{(m)}$ by

$$\lambda^{(p)} = (\lambda_1^{(p)}, \ldots, \lambda_{s_p}^{(p)}) \quad (p = 1, \ldots, m-1), \quad \lambda^{(m)} = (\lambda_1^{(m)} + 1, \ldots, \lambda_{s_m}^{(m)} + 1),$$

then there is a one-to-one correspondence between conjugacy classes in W and m-sets of partitions $(\lambda^{(1)}, \ldots, \lambda^{(m)})$ of n with $\sum_{p=1}^{m-1} \sum_{q=1}^{s_p} \lambda_q^{(p)} + \sum_{q=1}^{s_m} (\lambda_q^{(m)} + 1) = n$ (see [8]).

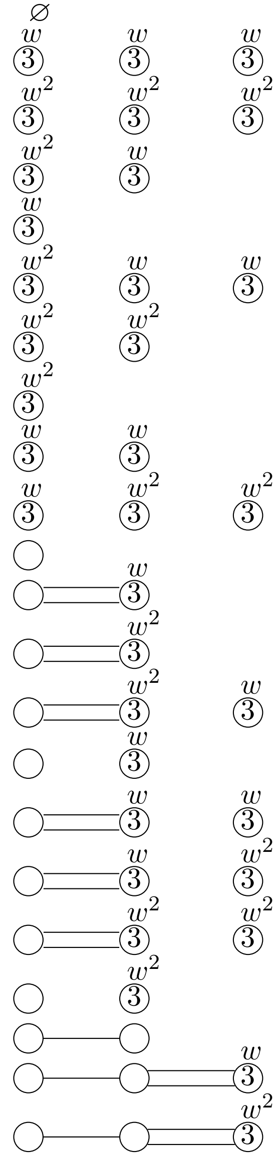
If $m = 1$, then $W = W(A_{n-1})$ (a Weyl group of type A_{n-1}), and if $m = 2$, then $W = W(C_n)$ (a Weyl group of type C_n), and so by putting $m = 1, 2$ in Theorem 3.3, we recover the results of Carter [5].

The admissible diagrams given in Theorem 3.3 are not the only ones which can be taken. We know that W contains a reflection subgroup $G(m,m,n) = W(D_n^m)$ (see [6]), and so D_n^m is an admissible diagram for W. However since the admissible diagrams given in Theorem 3.3 are in one-to-one correspondence with the conjugacy classes of W, we do not need the remaining part of the proof.

As an application of Theorem 3.3, we now give the following example.

Example 3.5. Consider the group $G(3,1,3)$. Then we have

Cycle type	Conjugacy class	Admissible diagram
$[1\ 1\ 1]$	1	$\varnothing$
$[1^{\xi}\ 1^{\xi}\ 1^{\xi}]$	$w_1w_2w_3$	
$[1^{\xi^2}\ 1^{\xi^2}\ 1^{\xi^2}]$	$w_1^2w_2^2w_3^2$	
$[1\ 1^{\xi}\ 1^{\xi^2}]$	$w_1^2w_2$	
$[1\ 1\ 1^{\xi}]$	w_1	
$[1^{\xi^2}\ 1^{\xi}\ 1^{\xi}]$	$w_1^2w_2w_3$	
$[1\ 1^{\xi^2}\ 1^{\xi^2}]$	$w_2^2w_3^2$	
$[1\ 1\ 1^{\xi^2}]$	w_1^2	
$[1\ 1^{\xi}\ 1^{\xi}]$	w_2w_3	
$[1^{\xi}\ 1^{\xi^2}\ 1^{\xi^2}]$	$w_1w_2^2w_3^2$	
$[2\ 1]$	(12)	
$[2^{\xi}\ 1]$	$(12)w_2$	
$[2^{\xi^2}\ 1]$	$(12)w_2^2$	
$[2^{\xi^2}\ 1^{\xi}]$	$(12)w_2^2w_3$	
$[2\ 1^{\xi}]$	$(12)w_3$	
$[2^{\xi}\ 1^{\xi}]$	$(12)w_2w_3$	
$[2^{\xi}\ 1^{\xi^2}]$	$(12)w_2w_3^2$	
$[2^{\xi^2}\ 1^{\xi^2}]$	$(12)w_2^2w_3^2$	
$[2\ 1^{\xi^2}]$	$(12)w_3^2$	
$[3]$	$(12)(23)$	
$[3^{\xi}]$	$(12)(23)w_3$	
$[3^{\xi^2}]$	$(12)(23)w_3^2$	



The elements in column 2 are representatives of the conjugacy classes of $G(3,1,3)$.

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