Abstract. The technique of perspective drawing has evolved over the past 600 years. Yet this has been primarily an external approach to visualizing and has lead to a mechanical interpretation of space. The influence of the Cartesian coordinate system has further forced perspective drawing into a rigid response to one’s surroundings. In the domain of creating architecture, this mechanical approach has led our culture into a banal and characterless environment. Furthermore, it has suppressed our emotions and crippled our intuition. Through the collaboration of our thesis for masters of architecture, we developed an approach to transforming space that begins immediately with one’s feelings. With the use of our method called the reverse engineered perspective, we have created a successful mathematical model that can be used to unfold one’s inner vision into physical space. The process takes the form of an intuitive sketch, from which a plan can be extracted by reversing the sequence of conventional perspective. Our case study focused on creating a chapel and extending inner geometries into the surrounding outdoor spaces. With this method of perspective art, drawing one’s inner feelings provides us with new insight into Christopher ALEXANDER’s theory of wholeness and our unfolding surroundings.

Key Words: perspective geometry, representation of art in architecture (Claude MONET: ‘The Bridge over the Water Lily Pond’, 1900), BRUNELLESCHI, ALBERTI, C. ALEXANDER and wholeness, N. SALINGAROS, B. MANDELBROT, morphology, projective geometry, proportions, transformations, reverse engineering.

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1. Introduction

Drawing can be perceived as an act in forming harmonious relationships between lines and shapes. At any point in time, the arrangement of lines and shapes forms a whole. The phenomenon of wholeness, in a drawing, occurs when individual geometries begin to interact with each other forming harmonious relationships.
Through the writings of architect and mathematician Christopher Alexander, he has described wholeness in relation to architecture and the design process. In *The Nature of Order*, Alexander defines wholeness as a *collection of centers working together to form an overall configuration of a place or thing*. He describes wholeness as a state of perception, through a ‘broad gestalt’ or ‘sweep of a figure’. Wholeness can be experienced emotionally through our perception of space. It can also be studied analytically; because a center or part, which is the focal point(s) of any whole, can be identified diagrammatically. With the desire to create a whole while designing, drawing becomes an essential tool for the design process.

If wholeness is somehow connected to our visual perception, as designers and architects, we presently have two methods available to us to articulate the feeling of the space we are designing; the conventional perspective and the conceptual sketch (natural perspective). The conventional perspective is constructed following the development of a plan and section. The natural perspective sketch is a rough idea, usually done in some perspective view. It is free of the constraints of a developed plan, and section.

The aim of this paper is to present a case study which explored a synthesis between drawing and wholeness, in which geometry of a place can inspire a context sensitive design. This paper follows a design process which a working tool called the *reverse engineered perspective* (REP) was used to delineate a plan from a ‘natural perspective sketch’. This process is in contrast to the conventional perspective in its approach and results of the design process.

2. The Renaissance: Development of the Conventional Perspective Drawing Method

Wholeness is both an internal and an external phenomenon. The Renaissance was a period when the external appreciation of wholeness was being written about and studied mathematically as well as artistically. That age presented an opportunity to revitalize the loss of the mathematical discipline and unite math with art through the discovery of perspective drawing. Many credit Brunelleschi as the pioneer who discovered the linear perspective view. It has been said that he did this by using mirrors and a viewing hole to create a precise drawing of the Baptistry in the Piazza del Duomo in Florence. Seemingly a complicated way to create a precise drawing, it forced the viewer’s eye to be fixed at the desired vanishing point.

The linear perspective painting by Brunelleschi was a rationalization of the idea that the location from which one views a building can have an impact on the viewer [7, 148–151]. Perspective drawing was explored further by Brunelleschi’s apprentice Alberti, who sought a method by which perspective drawing could be used to aid the process of architectural design and visually perceive one’s design in context. Alberti’s method was specific to the process of design, however it required developed architectural elevation, plans, and sections in order to construct the perspective drawing. His method incorporated two-dimensional drawings and angled them into perspective elevations by means of foreshortening the ground plane. Alberti began to use this in his own design process, “to work out the relationships between geometric lineament and volumetry of a building” [10, p. 28]. One can see the result of Alberti’s method in his buildings, such as Santa Maria Novella in Florence. Although the

\[1\] In *A Foreshadowing of 21st Century Art* [2], Alexander describes in great detail how wholeness can be experienced and analyzed through his study of ancient carpet patterns. Alexander offers an analysis of ancient Turkish carpet patterns, which he calls “a kind of miniature version of wholeness” [2, p. 31]
building is beautiful, it’s wholeness was derived from the building’s own inherent geometries and form, as an object, not necessarily from the immediate surrounding context.

Since ALBERTI, the fundamentals of the conventional perspective method has become refined; the development has enabled architects to construct perspectives with both a visual one-point and perspective views by mean of a direct perspective projection \(^2\). While different techniques might vary, this method of perspective drawing still have to be derived from already made plans or established vanishing points. As conventional perspective exists today, it is not a very good tool for discerning the emerging whole of a design as defined by Christopher ALEXANDER. The focus tends toward the correctness of the rigid system and not the emerging design. In addition, the Cartesian coordinate system has reinforced the notion that we can plot our environment through a coordinate system removed from its context \([9, 7.9]\). The fact that objects can be plotted in space creates a deep disconnect between abstract space and context. And in the computer age, the homogeneous model space in Computer Aided Design (CAD) further removes our innate human emotion in responding to the whole \([10, p. 149]\).

3. Impressionism: Development of the Natural Perspective Drawing Method

The natural perspective sketch, familiar to most designers as the ‘conceptual sketch’, has its artistic roots in the Impressionist art style. Impressionism, both in content and style, was an art of industrialized, urbanized Paris. Historians describe Impressionism as a style because of the collective exhibiting by its practitioners during the 1870s and 80s \([6, p. 905]\). But Impressionism was more than a stylistic movement; it was a group of artists who sought to illustrate scenes in a new way to communicate their impression of a place\(^3\). Impressionism was not concerned with vanishing points or correctly established grid lines, unlike preceding art movements, impressionist painting was the incorporation of all the great qualities of a free hand sketch: abbreviation, speed, and spontaneity \([13]\).

An example of Impressionist painting that best illustrates the idea wholeness with the natural perspective sketch was the series of MONET’s Japanese Bridge paintings (Fig. 1). By drawing the bridge at various times of the day, he showed an overriding broad gesture, the wholeness of the bridge and pond. MONET conveys the emotion he felt for his bridge and his garden throughout time. Although slight changes occur, the overall impression remains \([11, p. 111]\). In the diagrammatic sketch (Fig. 2), the centers or ‘individual geometries’ form together to give the painting a feeling of wholeness. The composition emerged as MONET drew the individual centers; the pond, the two flanking banks, the lilies, the hanging vines, the arch of the bridge, and the space between the pickets. MONET’s relaxed, syncopated style, which was typical among Impressionist painters, gives us a glimpse into the wholeness of his garden and pond at his home in Giverny.

MONET’s work is one example of the natural perspective drawing type. His paintings focused on capturing the overall feeling, and not necessarily the details. In other words, it was an active process in which the drawing was based on drawing the visual scene, the whole. Dr. BARNES observed this quality when describing how impressionist painters illustrate certain

\(^2\)Readers who are unfamiliar with the various conventional perspective methods should turn to Chapter 8 in Francis D.K. CHING: Design Drawing. Wiley and Sons, New York 1998.

\(^3\)The idea that feeling in a painting can generate feeling in the observer is one of the key concept of wholeness; “What matters is that the building” the room, the canyon, the painting, the ornament, the garden “as they are created, send profound feeling back towards us” \([3, p. 372]\).
quantities that are in relation to one another. Understanding how parts are interconnected to form a whole takes a while to process in the mind [4, p. 26–31]. The natural perspective as a tool in early stages of design is used to perceive the initial vision of a design. The whole, at this point in the process, is possibly more closely related to how we actually see and feel our design wants to be. Whereas conventional perspective at the later end of the design is concerned with discerning the parts from decisions based on forming a picture through developed plans and elevations.

4. Drawing Wholeness with the Natural Perspective Method

In the design process the use of natural perspectives can be an insightful in a way to inspire decisions. This case study features a process in which the site analysis of Sugarloaf Mountain became an integral part of exploring the design.

Sugarloaf Mountain is one of the largest mountains in the Washington, DC, metropolitan area. It’s a special place and has been had an impression on people since the early American settlers. The name Sugarloaf came from the first British troops who thought that the mountain’s shape looked like a 'sugar loaves' 4. The analysis was produced through a series of natural perspective drawings, drawn to observe the surroundings and capture the wholeness of the site, illustrating both daily and seasonal changes (Figs. 3 & 4). These site paintings were used during the design process to sketch overtop and explore various designs interventions Figs. 5 and 6.

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4http://www.sugarloafmd.com/
5. Reverse Engineered Perspective (REP): Extracting a plan from a Natural Perspective Drawing

Following the site analysis, the first design decision was to create a sequence of spaces that would eventually lead up the chapel. There already existed a path up the site but it was in need of repair (Fig. 5). It was also a long, steep climb up to the site, and at the top seemed like a logical place to design a bench, and a place to rest and to view the scenery. The location for the bench was as simple decision, there existed a tree forming a natural arching canopy, and provided a balanced tapestry of light and shade. The tree was drawn first, then a path to the bench. A bed of pink flowers around the tree formed a playful gesture, and informed the decision to add color and ornament to the steps giving the design scale Fig. 6.

5.1. Criteria for creating a natural perspective

Criterion 1: Identify places around the site which are inspirational from which to develop your natural perspective.

Criterion 2: Choose a center of focus, and begin to draw the scene, identifying natural gesture from the site, such as; trees, rocks, change in the topography, and the shape of the sky.

Criterion 3: Begin to draw your design intervention into the scene.

5.2. Sequential Steps for using the reverse engineered perspective

Now we turn to the question of how to build on these natural perspectives to give formal order to the sketch and generate a plan. The method of REP is implemented as followed Fig. 7:

Step 1: Within the natural perspective locate the station point (S.P.) or observer’s position (Fig. 7a).

Step 2: Locate the horizon line (H.L.) and central axis (Fig. 7b).
Step 3: Establish the ground line (G.L.) based on things drawn in the sketch, such as people, benches, tree, etc. Mark the boundaries of the sketch with vertical and horizontal lines (Fig. 7c).

Step 4: In order to find the picture plane (P.P.) one must know the distance (x), between the station point and the picture plane. In order to solve for x, we will use a maximum allowable line of sight, or cone of vision, that the human eye can see within (Fig. 7d). It has been determined that this falls within 30 degrees above a horizontal line of sight, and 30 degrees below a horizontal line of sight, forming a total cone of 60 degrees [5, 26–27].

By using a 1.68 meters scale factor, which is the average sight line of a human being, we can establish (y) by measuring in 1.68 metric increments, starting from the H.L. to the top vertical boundary of the drawing, this gives you the distance (x) between the S.P. and the P.P.

Using $\tan \theta$ and solving for $x$, the distance between station point and the picture plane is now obtained \(^5\) (Fig. 8):

$$\tan \theta = y/x, \quad x \tan \theta = y, \quad x = \frac{y}{\tan \theta}, \quad y = 6.99 \text{ m}, \quad x = \frac{6.99}{\tan 30} = 12.10 \text{ m} \quad (1)$$

\(^5\)Please note that the early development of the REP method has revealed the possibility of a potential error in determining the distance ($x$). By the very nature of how the natural perspective is drawn, the height at the top of the sketch can be a distorted approximation of the viewers station point (S.P.) to the picture plane (P.P.). Therefore, the scale measurement of 1.68 meters to the top boundary of the sketch is close approximation. Until further developments are made with the REP there are two remedies for this error; (if drawn at the site) one can physically measure your location (S.P.) and the distance to the object touching the (P.P.) to obtain the distance $x$. The other remedy, in order to obtain the correct scale, would be to proportionally adjust the plan as one extracts and develops the spaces within the sketch.
Figure 7: REP steps
**Step 5:** From the S.P. measure vertically $x = \text{(12.10 meters)}$, and draw a horizontal line for your P.P. Slide up both the P.P. and S.P. to the upper portion of the page to give yourself room to construct a plan (Fig. 7e, f).

**Step 6:** Locate the vanishing points (V.P.) on the H.L. by extending the lines from the object drawn in the natural perspective. Extend the V.P. on the H.L. to the P.P.; by connecting these points back to the S.P. this now gives you the angle to draw in plan (Fig. 7g, h).

**Step 7:** Locate two points from the object drawn in the natural perspective and extend these lines orthographically into the P.P. Now connect these points to the S.P. (Fig. 7i, j).

**Step 8:** Take the true angle found in step 6 and at the intersection of the angle and the lines in step 7 where they intersect is the true line in plan taken from the perspective (Fig. 7k).

**Step 9:** Repeat steps 6–8 and located each object set-by-step. Slowly a plan will begin to emerge from the perspective (Fig. 7l, m).

Extruding a plan from a natural perspective with the REP is a different approach to designing in the visual space than what is typically taught and practiced in design. The benefit in this approach is the ability to design, shape, and respond to what you can directly see. It is also important to keep in mind that reverse engineering a plan from a natural perspective is a continuous process of seeking out and revealing wholeness. As one seeks to generate a plan from a perspective, the emerging drawing begins to suggest areas to develop further, which might not be exactly clear from the perspective. In geometrical terms this is referred to as emerging properties and usually leads to differentiating elements in order to evolve a given structure\(^6\). Therefore the REP process reaffirms how the initial natural drawing is insightful into the potential feeling of the design.

\(^6\)As the plan emerges from the REP method, it's important to note that in our case study, the plan arrangement corresponding 'exactness' to the sketch (Figs. 10–11). Proportional adjustments to the plan will inevitably have to be made while designing. The power of the REP method is that it will help to reveal, not only the ‘centers’ within the natural sketch, but provide insightful ways for making decisions on what to develop further in plan, and what to keep from the natural perspective. Christopher Alexander also mentions these transformations will occur throughout the design process when one's trying to bring form a design in accord with wholeness. Plan development should be based on decisions that will ‘enhance’ the wholeness found in the natural sketch [3, p. 262–263].
Figure 9: The generated plan from the natural perspective shows how the look of a place can come prior to the formation of a plan. The natural perspective alone provided valuable insight into the arrangement of the plan and its wholeness.

6. Designing with the REP Method:  
The Design of a Chapel at Sugarloaf Mountain

The second part of the case study features the design of the chapel as developed with the REP method (Figs. 10–11). The development of the chapel intervention into the landscape occurred through a series of natural perspective drawings. When the design reach a harmonious fit, with the site, a plan was extracted from the perspective using the REP method following steps similar to Fig. 7.

The REP tool presents the ability for a designer to be inspired by the site. Getting the view right is essential, and quickly follows is the shaping of the design’s geometry together with the site’s geometry. The chapel at Sugarloaf illustrates this site inspiration by the result of its form language. The stone work for the small side chapels were a result of the topographical features of Sugarloaf. As well as the placement of these side chapels; governed by the surrounding trees Fig. 10. Also noticeable is the delicate trellis system hovering over the space. This system was derived from close observation of the forming of the trees and branches at the site Fig. 11. These design features were not simply aesthetic decision, but are a magnification of the large form [8].

The REP as a tool gives some freedom to a design to evolved base on its fitting into the context. It usually takes a couple of iterations of natural perspectives until the design fits with the existing context. Here the adaptation of the design comes in the wholeness of the form language with the surrounding space of the site.

7. Extended wholeness of Sugarloaf Mountain through drawing

Our case study has direct implications on planning in architectural design Fig. 12. The planning, as a whole, uses feeling and observation, which are directly interwoven through the unfolding drawings. Throughout this article we have focused closely on the shaping of center
by means of drawing intuitively and the inherent mathematical properties in these sketches that allows us to use REP to extract a plan. We would like to conclude with this analysis of the whole to prove that the whole was not just preserved but enhanced. As we look back, it was this reverse perspective process, starting with the path to the bench, and to the chapel, which informed each succeeding step.

Salingaros writes that the increase of wholeness depends on three aspects of the form that has been created: the small scale, the large scale, and the linking of the two together [12, p. 105]. For example, the existing centers that formed wholeness ‘A’ were the path up Sugarloaf Mountain, the bed of flowers, the tree overlooking the summit, the small hill, and the large rock. Wholeness ‘B’ was each of the proposed drawings: the path, the bench, and the chapel. The smaller connective geometries formed the harmonious link between the existing and the proposed: the added ornament on the individual steps, the color, and even the trellis in the chapel.

A dynamic design process is one where each intuitive drawing would be aimed at extending the existing environment. Throughout this case study it was crucial to pay close attention to each stage of development and to watch patterns develop. Small touches add to the geometric unity and visual interaction. It’s the weaving of centers throughout an existing site that begins to root a manmade structure into existence [1, p. 21].
Figure 12: Illustrates the enhanced wholeness, through drawing and the REP as an integral part of the design process. The new path, bench, and chapel emerged out of the existing context of Sugarloaf mountain

8. Conclusion

In conclusion, drawing as insight into wholeness is about understanding the nature of our environment in a way that is intertwined with in the design process. Using the REP method as a design tool is one way to working with the site directly, while one can respond immediately to the emotion found in the sketch. Getting the view right is essential, articulating the object and landscape together follows. As a sketch unfolds, one can be inspired by shapes and colors found at the site, and begin to build off of these things by extending geometries and harmonies from the site into ones building. The REP uses mathematics in the form of trigonometry and geometry. The result is that mathematics becomes more dynamic when used in unison with the process of architectural design, transcending it’s typical role of post analyzing the static state of architecture.

This article focused closely on the shaping of geometries by means of drawing intuitively and using the REP to extract an architectural plan from these intuitive drawings. And there are exciting possibilities with this model in practice; how to draw emotion from a city, from a small community, from a group of people, or from a individual client. However further exploration is still needed into the method of REP as a potential design tool, and its implication of the planning process of design. The hope would be that with the REP method, drawing can become a tool for gaining insight while one’s design continues to grows out of the existing context.

As humans we see, smell, touch, taste, and hear things, but do we really see, smell, touch, taste, and hear things? How do we see the earth and hear the voice of the ocean? Such ontological questions must be asked to fully engage and understand the things around us. If one does not think about the nature of a tree, one will never be part of a tree. We become part of a tree when we direct our consciousness toward it, in drawing form, and when we respond to it by the manner in which we dwell [13, p. 12].
References


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