Journal for Geometry and Graphics
Volume 28 (2024), No. 2, 199–212

Parameterized Folded State Shape Modeling of David Huffman's Ellipse

Aida Safary[1], Hamid Shafieasl[2], Jun Mitani[1]

[1] *University of Tsukuba, Tsukuba, Japan*
s2330170@s.tsukuba.ac.jp, mitani@cs.tsukuba.ac.jp

[2] *University of Utah, Utah, USA*
hamidmath2013@outlook.com

Abstract. This research aims to replicate David Huffman's Design with ellipses of two-degree two-vertices in the Grasshopper environment as a modeling tool that can be further explored as an architectural concept with various options to change basic geometric features of the shape. We used methods and approaches from previous studies on structural modeling to enhance Huffman's design with ellipses by implementing user input elements to change the shape parameters of the ellipse design.

Key Words: Geometry, CAD, Graphics, Curved Origami

MSC 2020: 51N05 (primary), 51M04, 51N15

1 Introduction

Curved crease origami is a challenging art form to study since it creates complicated and beautiful designs by bending the folds of a flat sheet of paper rather than making them straight. David Huffman, a computer scientist and artist, was one of the pioneers of curved crease origami, who experimented with the geometric and aesthetic potential of curved folds in the 1970s and 1980s. Huffman made many impressive models and designs with curved folds, using different curves, such as ellipses, parabolas, hyperbolas, and spirals [1]. However, Huffman did not share his work or his techniques, and his curved crease origami was largely undiscovered until his death in 1999. After that, researchers began to uncover and reconstruct his models, revealing the mathematical and artistic beauty of his designs [1].

The main idea of this research is to recreate and implement Huffman's design with ellipses (Figure 1) as an architectural element for any design purposes, either exterior facade design or interior design ornament, by considering them as dynamic elements that are sensitive to the light and can be stretched or contracted automatically, similar to the state of being folded or unfolded.

ISSN 1433-8157/

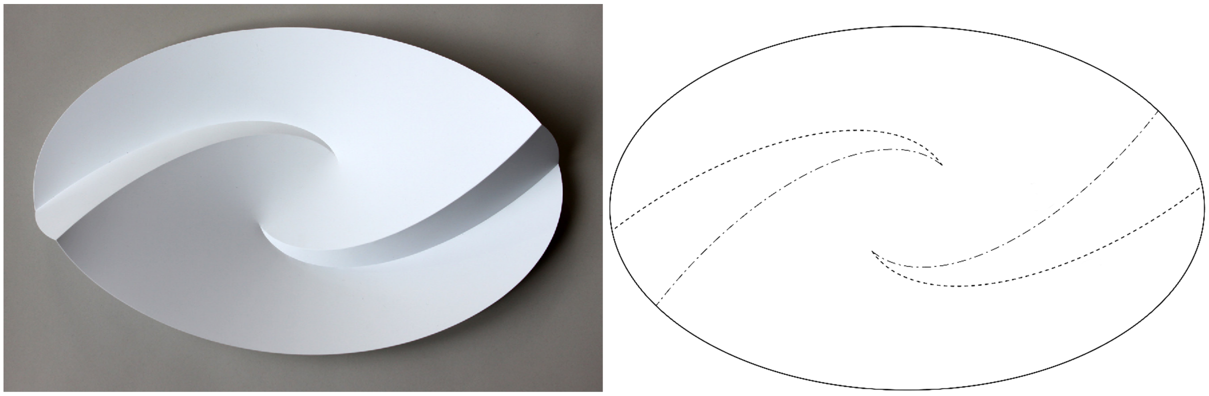

Figure 1: Huffman's Design with ellipses. Left: 3D state of the origami. Right: Crease pattern [1]. Dashed lines represent valley folds, while dotted lines represent mountain folds.

(a) (b) (c)

Figure 2: Ellipses as various elements, from left to right: Facade element (a) Pavilion (b) Interior element (c). Sketches are drawn by the author.

Figure 2 depicts some examples of the concept of using the ellipse structure in various architectural applications.

The design we concentrate on is a 3D origami model of ellipses of two degrees—two vertices (meaning there are two vertices where the two fold lines connect), which Huffman used to make a curved-fold structure that can change from a flat sheet to a 3D shape. We try to restore Huffman's model and reveal the mathematical elegance behind his artistic work. We applied a similar method to the additive algorithm method, in which the entire origami is represented as a discretized model using a set of planar quadrilaterals. The method uses a seeded quad strip as a starting point and builds the shape by sequentially adding quad strips [3]. We also included user input features in our model, enabling the user to adjust the parameters and constraints of the ellipse design to be able to see the change in basic geometric features and the final 3D shape of the model. We show the aesthetic and functional potential of Huffman's design and investigate the opportunities of creating new shapes and forms with curved creases.

2 Related work

In origami, the term "curved folding" describes the process of folding along curved lines as opposed to straight ones. Unlike traditional origami, which mostly uses straight creases, this style of curved folding makes the bending part of the surface build complicated and often more organic structures. With this technique, flat sheets of material may be used to form intricate, three-dimensional shapes [5]. Curved origami has attracted many artists because of the beauty of its form, and attempts have been made to elucidate its geometric properties. On the other hand, its useful engineering aspects have also attracted attention. Structural integrity and utility can be achieved by creating structures that can be folded compactly and then deployed into their desired shapes [6]. This methodology creates new opportunities to explore the aesthetic and functional potential of origami and to create deployable and self-assembling devices [7]. One notable work is Curve-Quad, an origami quadruped at centimeter scale [4]. The project's objective was to create a little robot that could crawl, drive, and fold itself using curved creases and a single actuator. The outcomes demonstrated that the Curve-Quad robot could self-fold, walk forward, and steer toward a light source. To comprehend the basic ideas of curved structures and how they are generated under geometrical constraints and energy considerations, researchers on "Folding on the Curve" investigated the geometric limitations and energy minimization in curved crease origami [2]. Another study concentrated on developing curved origami for tunable flexibility [8] achieved in the construction of origami structures with adjustable stiffness based on the function of tunable flexibility.

Tachi et al. carried out another important study on composite rigid foldable curved origami structures [10]. These structures are discretized by a set of flat quads rather than a curved surface model. The purpose of the research was to design multilayered rigid-foldable and flat-foldable vault structures with a curved-folding style. The result of the study was to produce a family of tubular structures that rigidly fold and unfold as compact deployable structures that can have significant implications for creating deployable architectural or other large-scale applications. According to the results, curved origami structures could offer a new range of stiffness-to-flexibility ratios, which is particularly useful in robotics and other fields requiring adaptable materials. A design method to approximate curved surfaces using generalized Miura-ori units was proposed by [11]. By enabling the creation of complex 3D structures from 2D sheets, this method advanced the field of mechanical metamaterials and opened up new useful applications. In the field of architecture, curved origami is used in a variety of designs, including the One-Fold project by Patkau Architects, Tal Friedman's Origami Pavilion, Zaha Hadid's Arum Pavilion, etc.

3 Preliminary

There are two main approaches to handling the shape of curved origami using a computer. The first approach treats the curved creases and surfaces analytically as continuous entities, and the second approach uses a discrete model that represents the curve as a poly-line and the surface as a quad-mesh. This section introduces the two distinct approaches.

3.1 The Geometry of Curved Origami with a Curved Crease

In the first approach, shapes involving curved folds are constructed based on findings in the field of differential geometry, using the relationships between the curvature and torsion of the folding curves in the 2D pattern and its folded 3D shape, as well as the folding angles, to form the 3D model. The relationship among these elements is expressed by formulas (1) to (3) [9]:

$$\kappa_{2D}(s) = \kappa(s)\cos\alpha(s), \tag{1}$$

$$\cot\beta_L(s) = \frac{\alpha'(s) - \tau(s)}{\kappa_{2D}(s)\tan\alpha(s)} = \frac{\alpha'(s) - \tau(s)}{\kappa(s)\sin\alpha(s)}, \tag{2}$$

$$\cot\beta_R(s) = \frac{-\alpha'(s) - \tau(s)}{\kappa_{2D}(s)\tan\alpha(s)} = \frac{-\alpha'(s) - \tau(s)}{\kappa(s)\sin\alpha(s)}, \tag{3}$$

where s is the arclength parameter, $\kappa_{2D}(s)$ is the curvature of the 2D projection of the folding curve, $\kappa(s)$ is the curvature of the folding curve in 3D, $\alpha(s)$ is the folding angle, $\tau(s)$ is the torsion of the folding curve in 3D, and $\beta_L(s)$ and $\beta_R(s)$ are the angles between the rulings and the tangent vector in the 2D projection on the left and right sides of the fold (Figure 3).

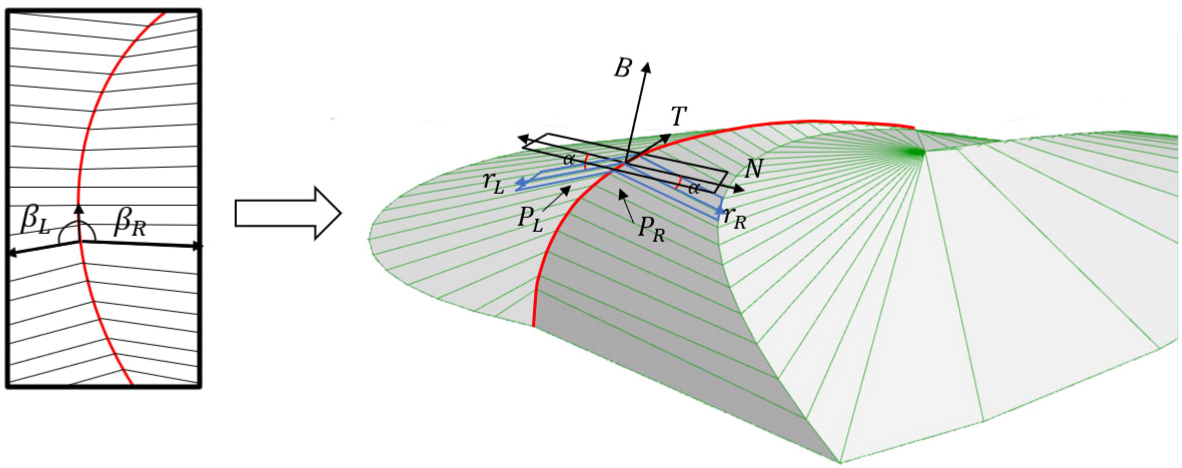

Figure 3: Geometry of curved folding.

When the folding angle α is constant, the torsion and curvature are related through the cotangent function as follows:

$$\cot\beta_L(s) = \cot\beta_R(s) = \frac{-\tau(s)}{\kappa_{2D}(s)\tan\alpha} = \frac{-\tau(s)}{\kappa(s)\sin\alpha} \tag{4}$$

From these relationships, it is theoretically possible to determine the 3D folding curve by specifying a single-variable angle function parameterized by the arclength for a 2D crease pattern. In this way, the 3D model can, in principle, be reconstructed. However, this is clearly limited to cases where there is only a single curved fold. Huffman's ellipse, which is the focus of this study, includes multiple curved creases and features the unique situation where the creases terminate within the interior of the region, making it challenging to analytically derive the resulting shape. Therefore, we adopt a discrete model, as described in the following subsection.

3.2 An Additive Algorithm Method for Origami Design

One method for constructing the shape of curved origami is to represent it as a discretized model composed of flat quads. In this approach, the geometric constraints are limited to ensuring that the quads forming the shape remain flat and that the sum of the sector angles around each interior vertex equals 2π. This makes it easier to construct the shape compared to the analytical approach based on the equations described in the previous subsection.

The additive algorithm for origami design [3] is an approach for creating complicated origami surfaces by increasingly adding strips of quadrilateral facets. This method simplifies the design process of a 3D origami surface by breaking it into manageable steps while preserving its geometric constraints.

First, a quad strip called the *seed* is prepared, and additional quad strips are sequentially added to it. The sequence of boundary edges where new quads are connected is referred to as the *growth front*. When the sequence of vertices constituting the growth front is denoted by $\mathbf{x}_i$, the edge extending from the vertex $\mathbf{x}_i$ connects to a newly generated vertex $\mathbf{x}'_i$. As shown in Figure 4, the vector $\mathbf{x}'_i - \mathbf{x}_i$ is represented as $\mathbf{r}_i$, and the problem reduces to determining the vector $\mathbf{r}_i$ from the growth front. Let $\theta_{i,j}$ $(1 \leq j \leq 4)$ denote the sector angle around the vertex $\mathbf{x}_i$, then the following equation holds:

$$\sum_{j=1}^{4} \theta_{i,j} = 2\pi \tag{5}$$

to ensure that the structure can be folded without tearing or overlapping.

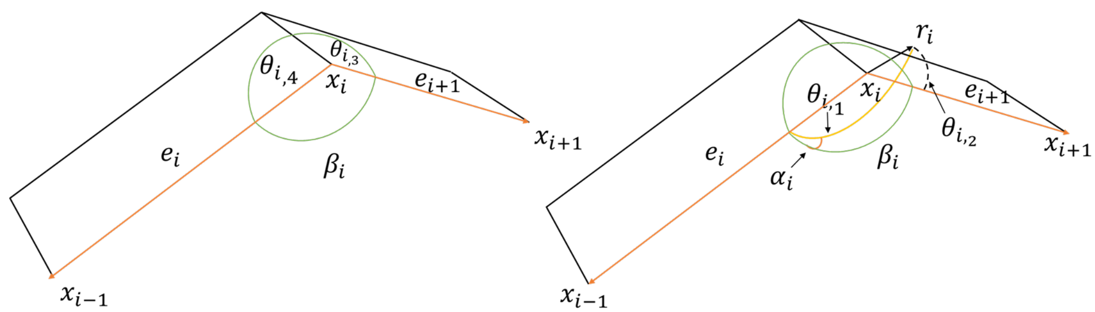

Figure 4: Construction process of the initial seed of an origami.

The direction of $\mathbf{r}_i$ is determined by the angles α_i and β_i. Here, β_i is the angle between the edges $\mathbf{e}_i$ and $\mathbf{e}_{i+1}$, while α_i, referred to as the *flap angle* in [4], is the angle between the plane containing β_i and the quad added via $\mathbf{e}_i$. Let $\theta_{i,3}$ and $\theta_{i,4}$ represent the two sector angles around $\mathbf{x}_i$ based on the seed quads, and $\theta_{i,1}$ and $\theta_{i,2}$ represent the two sector angles obtained from the newly added quads. Based on the spherical geometric relationships and the spherical law of cosines, the following equations hold:

$$\cos\theta_{i,2} = \cos\beta_i \cos\alpha_i + \sin\beta_i \sin\alpha_i \cos\theta_{i,4} \tag{6}$$

By considering Equations (5) and (6), we can calculate the values for $\theta_{i,1}$, $\theta_{i,2}$, Then, we obtain the following equations:

$$\theta_{i,1} = \tan^{-1}\left(\frac{\cos k_i - \cos\beta_i}{\sin\beta_i \cos\alpha_i - \sin k_i}\right), \quad \theta_{i,1} \neq \frac{\pi}{2} \tag{7}$$

and

$$\theta_{i,2} = k_i - \theta_{i,1}, \tag{8}$$

where $k_i = 2\pi - \theta_{i,3} - \theta_{i,4}$, and angle α_i is a user-specified parameter. By the calculation of $\theta_{i,1}$ and $\theta_{i,2}$ we can determine the direction of $\mathbf{r}_i$ which determines the next quads strip.

4 Methodology

In this section, we explain the process of creating Huffman's design with ellipses 3D origami structure as a discrete model incorporating triangles and quads. Since a triangle can be regarded as a degenerate quadrilateral where one edge collapses, the subsequent discussion will not distinguish between the two. We want to use a method where its idea comes from the additive method introduced in the previous section, and we make sure that it guarantees the developability, in other words, we ensure that for all interior vertices, the sum of their sector angles is 2π. The explanation process includes: assigning the boundaries, assigning and dividing the crease pattern into patches, folding them, and evaluating the angle condition around the vertices. Finally, we will explain the performance of our system implemented in the grasshopper environment.

4.1 Boundary and Crease Curves

To begin with, let's assign labels to the different parts of the unfolded plane as shown in Figure 5. *lls* and *rls* stand for left and right lower surfaces; *lus* and *rus* stand for left and right upper surfaces; *lms* and *rms* stand for left and right middle surfaces; *lts* and *rts* stand for left and right triangular surfaces; *llc* and *rlc* stand for left and right lower curves; *luc* and *ruc* stand for left and right upper curves; V_1 and V_2 stand for left and right curve tip vertices.

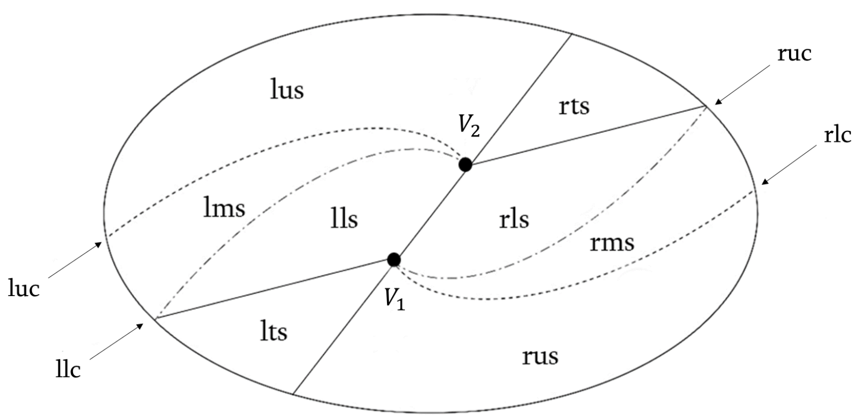

Figure 5: Abbreviation of names of each part.

An ellipse can be defined with parametric equations with parameter θ $(0 \leq \theta \leq 2\pi)$ which represents the angle that determines the Cartesian coordinates of the point through the following parametric equation.

$$\begin{cases} x = a\cos\theta \\ y = b\sin\theta \end{cases}$$

The bounding ellipse is defined by the values of a and b in this equation. To set the crease curves, the endpoints of the crease curve, V_1 and V_2, are positioned symmetrically with respect to the origin. Now we use copies of the smaller scale of four other ellipses for crease curves. With rotation and translation, we locate them to the V_1 and V_2 as can be seen in Figure 6(a). Besides, we assign two angle values ψ_1, ψ_2 as controlling parameters of rotations. As a result, the crease pattern shown in Figure 6(b) is obtained.

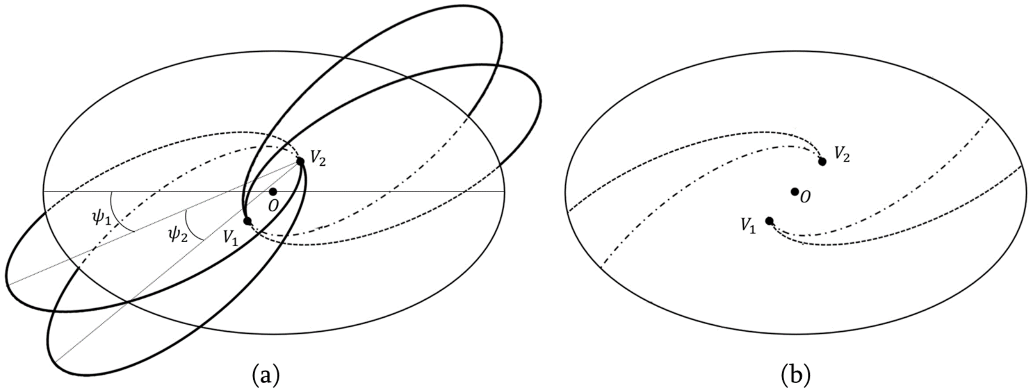

Figure 6: (a) The arrangement of ellipses used to obtain the crease pattern. (b) The resulting crease pattern.

4.2 Polygonization

Having determined the plane boundary and crease lines, we can go to the next step. Since we aim to create a 3D model that is a polygonal mesh, we represent a surface as a set of triangles or plane quadrilaterals that are assembled, and for each pair of adjacent polygons, a specific angle is allocated for their dihedral angle.

Here, we are going to describe the segmentation, i.e. polygonization. If we draw a line that passes through OV_1 and OV_2, we can see that the shape is symmetric to this line and therefore we only describe the division of the left side of the line. The right side would be the same.

First, a sequence of vertices is placed on *luc*. This is achieved by sampling the parameter θ of the elliptical equation, forming the *luc* at equal intervals. Let the number of vertices be N. Next, the same number N of vertices is placed on *llc* by dividing the parameter range of the *llc*'s elliptical equation into $N-1$ segments. Then, for each vertex on *luc*, the intersection points between the boundary ellipse and the straight lines passing through the origin (the center of the boundary ellipse) and the *luc* vertices are determined. These intersection points form the sequence of vertices on the boundary ellipse.

With these steps, the positions of all necessary vertices are determined. Subsequently, the region *lls* is divided by connecting V_1 with each vertex on *llc* (Figure 7a). Then, the region *lms* is divided by sequentially connecting the vertex sequences on the *llc* and *luc* from one end to the other (Figure 7b). Finally, the region *lus* is divided by similarly connecting the vertex sequence on the *lms* with that on the boundary ellipse (Figure 7c). The region *lts* is not divided. The same process is applied to the right side, completing the division of the entire structure (Figure 7d).

The folding process is only the assignment of the fold-angle for all edges. In the rest of this paper, we explain how do we assign the angle values among these edges while satisfying the constraint that the sum of the sector angles around each interior vertex equals 2π.

4.3 Seed and its Extension

According to the additive algorithm method described in Section 3.2, a seed refers to the initial folded configuration of a quad strip from which the design process begins. This seed serves as the starting point for growing a folded surface in a three-dimensional state. In our method, the positions of the initial triangle are first determined, and then the coordinates of

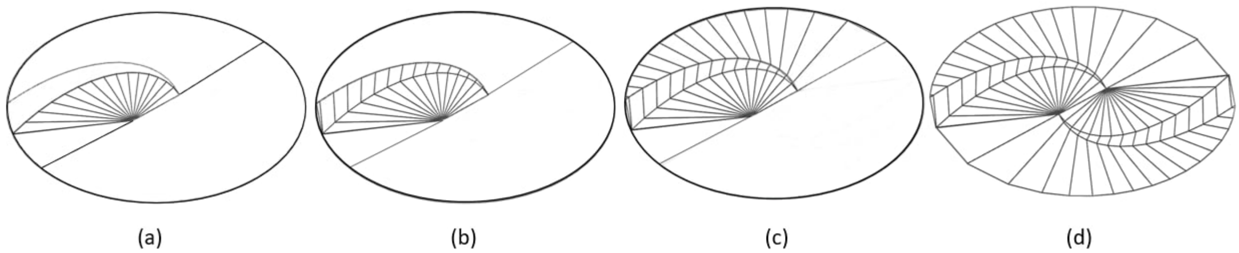

Figure 7: Dividing the crease pattern onto the regions, from left to right: inner regions, two main branches, outer regions, overall regions.

the vertices are sequentially calculated. According to the division that we indicated before, we choose four polygonal faces, s_1, s_2, s_3, and s_4, as shown in Figure 8 for the initial step.

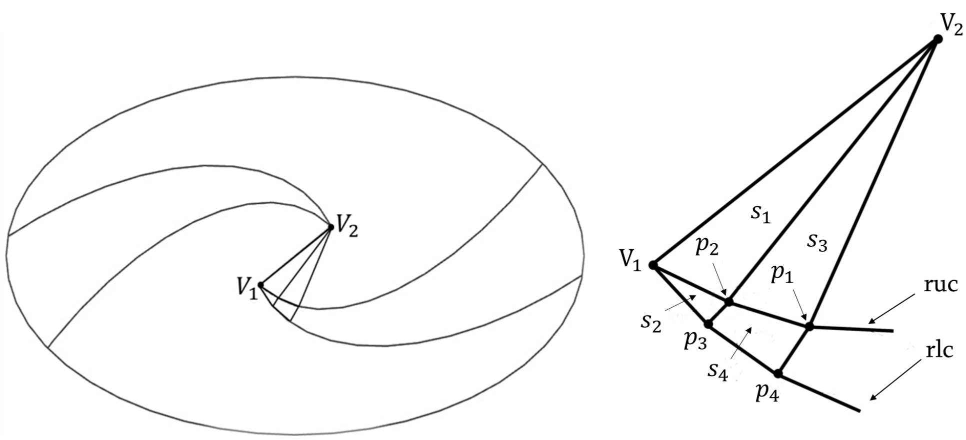

Figure 8: The first four polygonal faces were used in constructing the 3D shape.

We assume that the unfolded plane lies on the xy-plane such that the center of the ellipse coincides with the origin and the long diagonal of the ellipse aligns with the x-axis. For simplicity, we will use the same capital letters to denote the corresponding pieces of the folded state as we used for the unfolded state. For instance, we use P_i to denote a vertex of the folded state corresponding to p_i for the unfolded state. Consider Figure 9 and let ϕ be the angle between triangles $P_2V_2V_1$ and the xyplane specified by the user. Using ϕ, we perform a rotation on triangles, $V_1p_2V_2$ resulting in $V_1P_2V_2$ while keeping V_1 and V_2 fixing as depicted in Figure 9a.

Now for determining the position of P_1 and P_3 in Figure 9b, consider the below formula, which is described in [3].

$$\cos\theta_2 = \cos\theta_1 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha,$$

where α is the *flap angle* described in Section 3.2 and is one of the user-specified parameters. From the above formula, we get β as the following:

$$\beta = \cos^{-1}\left(\frac{\cos\theta_2\cos\theta_1 \pm \sin\theta_1\cos\alpha\sqrt{\cos^2\theta_1 - \cos^2\theta_2 + \sin^2\theta_1\cos^2\alpha}}{\cos^2\theta_2 + \cos^2\theta_1}\right).$$

By preserving the angle $P_1P_2V_2 = \theta_3$ and the length $P_1P_2 = p_1p_2$, we rotate the point P_1 about the line V_2P_2 to get the angle between edges V_1P_2 and P_1P_2 equal to β. P_3 is obtained

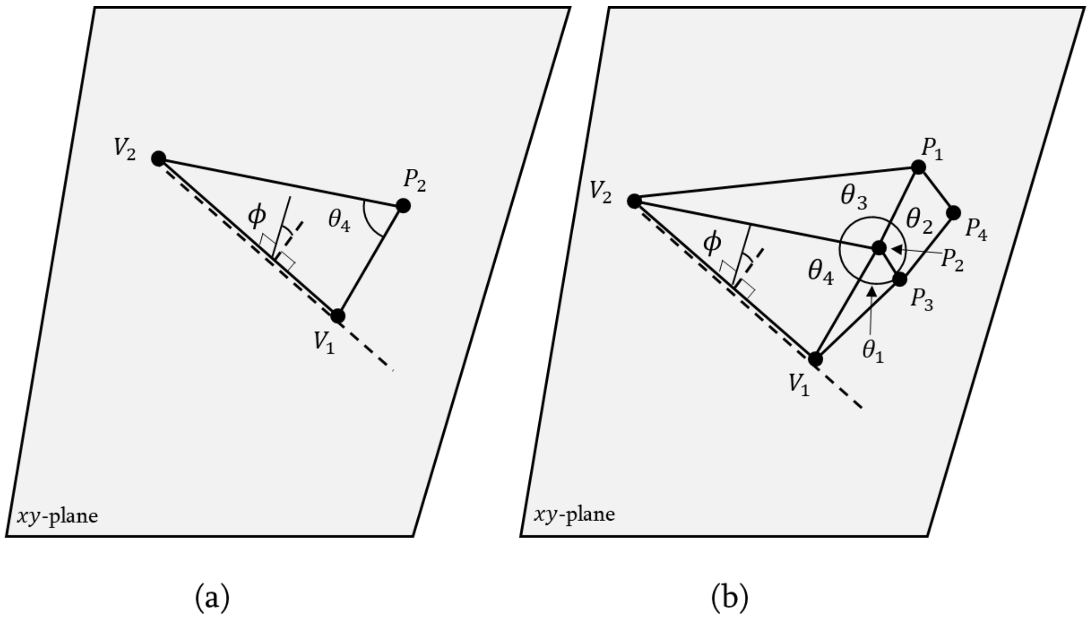

Figure 9: The initial steps for constructing the folded state.

by rotating it by α with respect to the β plane. In this way, the positions of P_1, P_2, and P_3 are determined. Since P_4 lies on the plane formed by these points, its position can be determined accordingly. The most notable characteristic of the surface at point P_2 is its fulfillment of the angle condition around the vertex, denoted by $\sum_{j=1}^{4} \theta_j$.

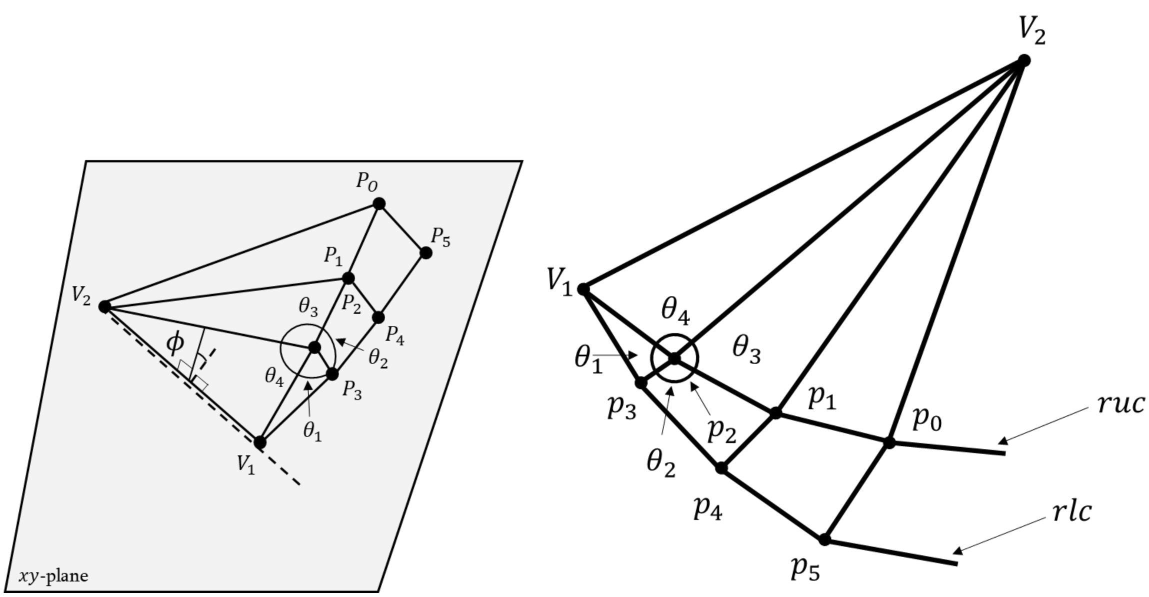

Figure 10: Left: Finding the position of a new point of P_4. Right: Extension of the initial seed.

Now we continue by applying the same method for the next point. Note that from this step on the flap angle, α is automatically obtained from the previous step. By assigning similar names for new points and angles, as depicted in Figure 10, and following the same argumentation, we could decide the folded state of the following polygons in both in *rls* and *rms*.

With this method, we are able to complete *RLS* and *RMS*. Besides, the same steps can be taken to complete LLS and LMS as well (Figure 11).

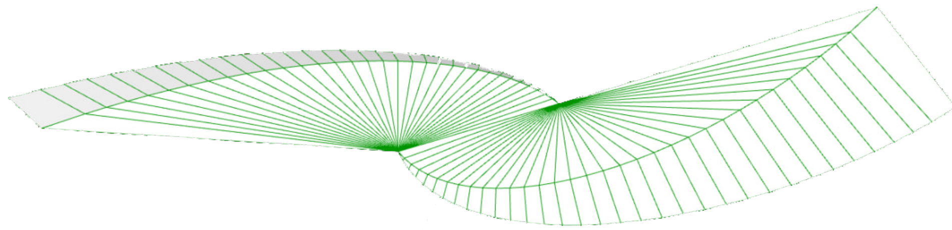

Figure 11: The 3D model of the constructed regions LMS, LLS, RMS, and RLS.

Subsequently, the shapes of the remaining regions RTS, RUS, LTS, and LUS are constructed. As shown in Figure 12a, let $\mathbf{u}$ be the vector along the edge at the boundary of region *lus*, and $\mathbf{v}$ be the vector along the first line segment of the curve *luc*. Let γ be the angle between $\mathbf{u}$ and $\mathbf{v}$ in the unfolded state, and δ be the angle formed by region *rts* around V_2. These angles remain unchanged in the folded state. When constructing the folded state of LUS, we first rotate the first quad s in the region *lus* around the vector $\mathbf{v}$ in the folded state so that these angles (γ and δ) do not change. The rotation angle that satisfies this condition is found using a binary search. In this way, RTS is determined, and the position of the first quad in LUS is fixed, and the remaining region of LUS is determined using the method described before. Finally, LTS and RUS are constructed using symmetry.

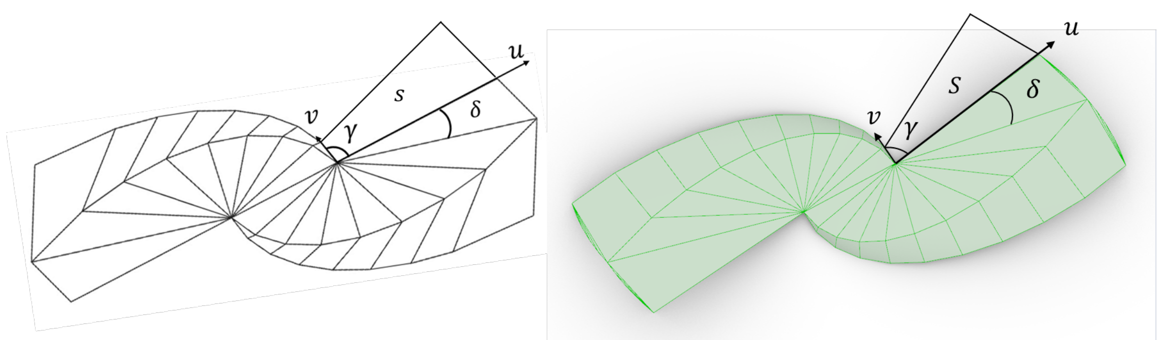

Figure 12: To ensure that the angle δ within RTS and the angle γ within the first quadrilateral of LUS remain unchanged, the rotation angle along the vector v is determined.

Using the above method, the whole of the 3D shape can be constructed as shown in Figure 13.

5 Result and Evaluation

The method described in the previous section was implemented in the Grasshopper environment on the 3D CAD software Rhino, resulting in the generation of a 3D model of Huffman's ellipse. Here, we present models generated by varying the parameters and provide an evaluation of their shapes. Subsequently, we introduce an example application.

Figure 13: The final constructed 3D shape.

5.1 Model Generation Using the Proposed Method

By adjusting the values of various parameters using slider bars, users can generate the 3D shape in real-time. Figure 14 shows the obtained 3D models, and Table 1 presents the numerical information for the models.

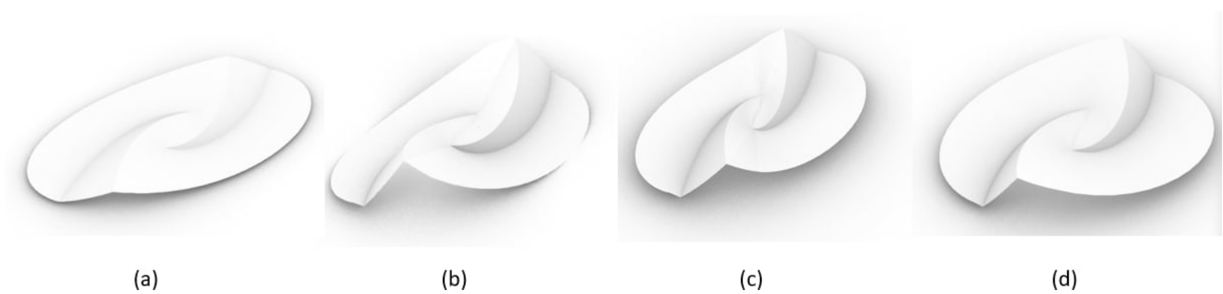

Figure 14: From left to right: figures that depict changeable parameters described in table 1.

The input parameters in Table 1 are as below:

- α: *flap angle* for the first triangle.
- ϕ: angle between the xy-plane and the first triangle surface.
- AR_{boundary}: aspect ratio of the boundary ellipse.
- AR_{sub}: aspect ratio of the smaller ellipses used for crease curves.
- ψ_1, ψ_2: rotation angles of the smaller ellipses used for the crease curves.

The geometric validity of the generated 3D shape is evaluated from the perspective of developability as the following metric.

- developability error: the maximum error value in degrees among the differences between the sum of the sector angles around each vertex 360°.

Table 1 lists the parameter values used to generate each model shown in Figure 14, along with the corresponding developability error values.

By modifying the parameter values, we were able to generate variations that resemble Huffman's ellipse in appearance but differ in crease arrangements, folding angles, and overall shape. When the values of α and ϕ are small, the resulting shape is closer to a flat configuration, whereas larger values of these parameters produce more pronounced folding angles and a highly three-dimensional structure. The sum of angles around each vertex is nearly 2π, and the deviation is small enough to be considered negligible during fabrication.

model	α	ϕ	AR_{boundary}	AR_{sub}	ψ_1	ψ_2	developability Error
a	23°	2°	0.60	0.48	45°	16°	1.50×10^{-5}
b	55°	10°	0.60	0.58	45°	25°	2.44×10^{-5}
c	40°	10°	0.60	0.30	30°	20°	1.98×10^{-5}
d	35°	6°	0.90	0.48	45°	16°	2.17×10^{-5}

Table 1: Parameters and error values.

5.2 Potential Applications of Huffman's Ellipse Origami

Huffman's ellipse origami has potential applications in the field of architecture, as suggested by the illustrations in Figure 2 in Section 1. Other example of such an application is shown in Figure 15, representing an idea we devised. By incorporating a mechanism that sensitively responds to light and airflow, the structure can flatten or fold into a three-dimensional form, adapting dynamically to environmental changes. The ratio of the area of the shadow of the folded state with respect to the area of the unfolded state is about 93%. Given this value, the effect on adjusting the amount of light entering a room can be considered minimal, suggesting its use in altering the room's ambiance. Alternatively, it may serve as a functional element as a pathway for air circulation (note that this is a conceptual proposal, and the mechanism for deformation has not been considered.) The proposed method for shape generation is useful for performing such visual simulations.

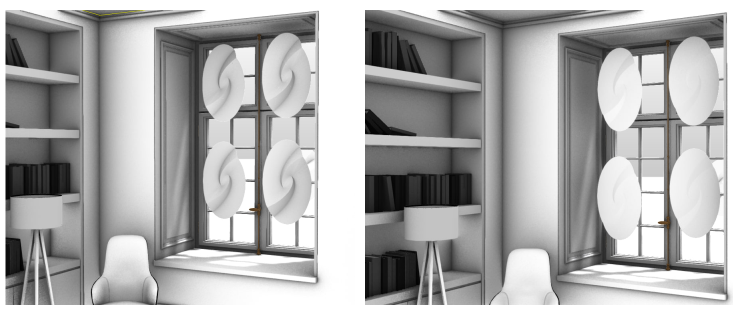

Figure 15: The change in visual impressions caused by the deformation of Huffman's ellipse origami arranged on a window.

6 Conclusion

In conclusion, we have developed a 3D digital module based on David Huffman's ellipse design with two degrees and two vertices. This interactive module empowers users to manipulate geometric features such as size and fold angle, offering flexibility in design customization. Additionally, the module is fully compatible with CAD environments, allowing it to be seamlessly integrated into architectural projects as a unique design element.

In this study, we focused on the shape of Huffman's ellipse; however, the proposed method can be applied to other curved origami designs as well. For example, it is straightforward to change the contour from an ellipse to other shapes, and variations in the crease patterns, such as their shape, arrangement, and number, can also be introduced. By further developing the proposed method, it becomes possible to construct a wide range of origami designs.

With the digital design phase now complete, we can have the prospect of creating a physical model in the future, which will further bring this innovative concept to life.

Acknowledgments

This work was supported by JST CREST Grant Number JPMJCR1911, Japan.

References

[1] E. D. Demaine, M. L. Demaine, and D. Koschitz: *Reconstructing David Huffman's Legacy in Curved-Crease Folding*. In *Origami 5: Fifth International Meeting of Origami Science, Mathematics, and Education*, 39–52. A K Peters/CRC Press, 2011.

[2] M. A. Dias, L. H. Dudte, L. Mahadevan, and C. D. Santangelo: *Geometric Mechanics of Curved Crease Origami*. Physical Review Letters **109**(11), 114301, 2012. doi: `10.1103/physrevlett.109.114301`.

[3] L. H. Dudte, G. P. T. Choi, and L. Mahadevan: *An additive algorithm for origami design*. Proceedings of the National Academy of Sciences **118**(21), 2021. doi: `10.1073/pnas.2019241118`.

[4] D. Feshbach, X. Wu, S. Vasireddy, L. Beardell, B. To, Y. Baryshnikov, and C. Sung: *CurveQuad: A Centimeter-Scale Origami Quadruped that Leverages Curved Creases to Self-Fold and Crawl with One Motor*. In *2023 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2485–2492. IEEE, 2023. doi: `10.1109/iros55552.2023.10342339`.

[5] T.-U. Lee, Y. Chen, M. T. Heitzmann, and J. M. Gattas: *Compliant curved-crease origami-inspired metamaterials with a programmable force-displacement response*. Materials & Design **207**, 109859, 2021. doi: `10.1016/j.matdes.2021.109859`.

[6] J. Mitani: *Curved-Folding Origami Design*. CRC Press, 2019. ISBN 9780429059179. doi: `10.1201/9780429059179`.

[7] D. Raducanu, V. D. Cojocaru, V. A. Raducanu, A. Nocivin, N. Serban, I. Cinca, E. M. Cojocaru, L. Moldovan, C. Trisca-Rusu, and I. V. Balkan: *Design and Optimization of a Curved-Crease-Folding Process Applied to a Light Metallic Structure*. Processes **9**(7), 1110, 2021. doi: `10.3390/pr9071110`.

[8] Y. Sun, K. Song, J. Ju, and X. Zhou: *Curved-creased origami mechanical metamaterials with programmable stabilities and stiffnesses*. International Journal of Mechanical Sciences **262**, 108729, 2024. doi: `10.1016/j.ijmecsci.2023.108729`.

[9] T. Tachi: *One-DOF Rigid Foldable Structures from Space Curves*. Proceedings of the IABSE-IASS Symposium 20–23, 2011.

[10] T. Tachi: *Composite Rigid-Foldable Curved Origami Structure*. Proceedings of Transformables 18–20, 2013.

[11] F. Wang, H. Gong, X. Chen, and C. Q. Chen: *Folding to Curved Surfaces: A Generalized Design Method and Mechanics of Origami-based Cylindrical Structures*. Scientific Reports **6**, 2016.

Received December 2, 2024; final form February 17, 2025.